Nonlinear Vibration and Instability Response of Embedded Pipes Conveying Viscose Fluid Using DQM

H. Rahimi pour[1], M. Ghaytani[2], R. Kolahchi[3]

In this paper, nonlinear vibration and instability response of an embedded pipe conveying viscose fluid is investigated. The pipe is considered as a Timoshenko beam embedded on an elastic foundation which is simulated by spring constant of the Winkler-model and the shear constant of the Pasternak-model. The external flow force, acting on the beam in the direction of the flexural displacement is described by the well-known Navier-Stokes equation. The corresponding governing equations are obtained using Hamilton's principle considering nonlinear strains and first shear deformation theory. In order to obtain the nonlinear frequency and critical fluid velocity for clamped supported mechanical boundary condition at two ends of the pipe, Differential quadrature method (DQM) is used in conjunction with a program being written in MATLAB. The effect of dimensionless parameters such as aspect ratios of length to radius of the pipe, Winkler and Pasternak modules, fluid velocity and viscosity as well as the material type of the pipe on the frequencies and instability of pipe are investigated. Results indicate that the internal moving fluid plays an important role in the instability of the pipe. Furthermore, the nonlinear frequency and instability increases as the values of the elastic medium constants and viscosity of fluid increases.

Keywords: **Nonlinear vibration, Instability, Fluid, DQM.**

1. Introduction

A vast variety of structures or structural components used in civil, mechanical, aerospace and defence engineering are pipes. These pipes are often used to store and transport high-pressure gases and liquids for various hydraulic applications. A good understanding of their mechanical behaviour, including vibration, bending and impulse response, is a must for the successful design and application of pipes in engineering practice. Vibration analysis of pipes has been studied by many researchers.

Pipes conveying fluid have become one of the important structures widely used in engineering, such as those employed in nuclear reactor, ocean mining, heat exchanger and drug delivery [1-3]. In such applications, one of the most important issues is to accurately measure the vibration characteristics, such as natural frequency, stability and critical flow velocity of the fluid-conveying systems. It is not surprising, therefore, that the study on this topic is constantly expanding in the past decades. Indeed, the vibration and stability of pipes conveying fluid have been studied for more than six decades, both theoretically and experimentally. A good review of the related literature was provided by Païdoussis and Li [4].

In general, methods of vibration analysis can be classified as analytical methods and numerical approaches. It is generally agreed that the procedure for analytical/closed form solutions of such vibration problems is tedious and in most cases, no such solutions might exist at all. This promotes the popularity of the numerical methods in solving this type of problems. One class of numerical approaches is differential quadrature methods (DQMs), which was first introduced by Bellman et al. [5] in 1972 for solving partial differential equations.

2. Formulation

Fig. 1 shows the pipes modeled as a Timoshenko beam with length L, inner radius r_1, outer radius r_2 and equal thickness h embedded in an elastic medium. The surrounding medium is described by the Winkler foundation model with spring constant k and Pasternak foundation model with shear constant G. Based on the Timoshenko beam theory, the displacements of an arbitrary point in the beam along the x-

1. Pars Oil and Gas Co, Iran, *E-mail: hrahimipour@pogc.ir*
2. NIOPDC, Iran, *E-mail: mehdi_mn28@yahoo.com*
3. University of Kashan, Kashan, Islamic Republic of Iran, *E-mail: r.kolahchi@kashanu.ac.ir*

and z-axes, denoted by U and W respectively, take the form of[6]

$$\tilde{U}(x,z,t) = U(x,t) + z\psi(x,t)$$
$$\tilde{W}(x,z,t) = W(x,t) \tag{1}$$

Where U(x, t) and W(x, t) are displacement components in the midplane, ψ is the rotation of beam cross-section and t is

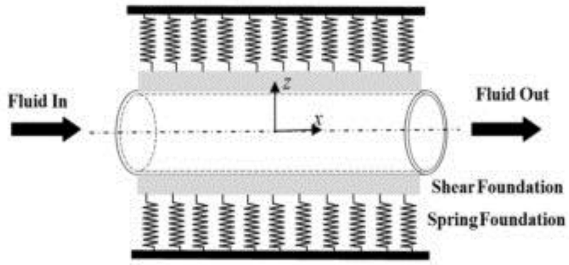

Fig. 1 Geometry of the pipe modeled as the nonlocal Timoshenko beam.

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$$\varepsilon_{xx} = \frac{\partial U}{\partial x} + z\frac{\partial \psi}{\partial x} + \frac{1}{2}\left(\frac{\partial W}{\partial x}\right)^2 \tag{2}$$
$$\gamma_{xz} = \frac{\partial W}{\partial x} + \psi$$

For a beam structure, the constitutive relations can be approximated to one-dimensional form as

$$\sigma_{xx} = C_{11}\varepsilon_{xx}$$
$$\sigma_{xz} = C_{55}\left[\frac{\partial W}{\partial x} + \psi\right] \tag{3}$$

where E and G are Young's modulus and shear modulus, respectively. The strain energy V of the pipes embedded in an elastic medium can be calculated from

$$U = \frac{1}{2}\int_0^L \int_{Ai} (\sigma_{xx}\varepsilon_{xx} + \sigma_{xz}\gamma_{xz})\, dA_i dx \tag{4}$$

Submitting (2) into (4) gives:

$$U = \frac{1}{2}\int_0^L \left\{N_x\frac{\partial U}{\partial x} + M_x\frac{\partial \psi}{\partial x} + \frac{1}{2}N_x\left(\frac{\partial W}{\partial x}\right)^2 + Q_x\frac{\partial W}{\partial x} + Q_x\psi\right\}dx \tag{5}$$

The normal resultant force N_x, bending moment M_x, and transverse shear force Q_x are defined as

$$N_x = \int_A \sigma_{xx}\, dA,\ M_x = \int_A \sigma_{xx} z\, dA,\ Q_x = \int_A \sigma_{xz}\, dA \tag{6}$$

The work done by the elastic medium is denoted by

$$\Omega = \frac{1}{2}\int_0^L \left(-K_w W + G_p \nabla^2 W\right) W dx \tag{7}$$

The work done by the viscose fluid is denoted by

$$m_f\left[\frac{\partial}{\partial t} + U_f\frac{\partial}{\partial x}\right]\left[\frac{\partial W_1}{\partial t} - U_f \sin\theta\right] =$$
$$-\nabla P.A_f + \mu A_f\frac{\partial^2}{\partial x^2}\left[\frac{\partial W_1}{\partial t} - U_f\sin\theta\right]$$
$$m_f\frac{\partial^2 W_1}{\partial t^2} + m_f U_f\frac{\partial^2 W_1}{\partial x \partial t}\cos\theta + m_f U_f\frac{\partial^2 W_1}{\partial x\partial t}$$
$$+ m_f {U_f}^2\frac{\partial^2 W_1}{\partial x^2}\cos\theta = -\nabla P.A_f + \mu A_f\frac{\partial^3 W_1}{\partial x^2 \partial t} \tag{8}$$
$$+ U_f \mu A_f\frac{\partial^3 W_1}{\partial x^3}\cos\theta + U_f\mu A_f\left(\frac{\partial^2 W_1}{\partial x^2}\right)^2\sin\theta$$

$$\Omega_{fluid} = \int (F_{fluid}) w dz, \tag{9}$$

The kinetic energy T is given by

$$T_{tube} = \frac{1}{2}\rho_t\int_0^L\int_A\left[\left(\frac{\partial U}{\partial t} + z\frac{\partial \psi}{\partial t}\right)^2 + \left(\frac{\partial W}{\partial t}\right)^2\right]dA\, dx \tag{10}$$

The equations of motion of the fluid-conveying pipes embedded in an elastic medium can be derived from the Hamilton principle

$$\int_{t_0}^{t_1}\left[\delta U - (\delta T + \delta\Omega)\right] = 0 \tag{11}$$

Introducing the following dimensionless quantities

$$\xi = \frac{x}{L} \quad (w,u) = \frac{(W,U)}{r} \quad \eta = \frac{L}{r} \quad \bar{\mu} = \frac{\mu}{r\sqrt{E\rho_f}}$$
$$\bar{I} = \frac{\rho I}{\rho A r^2} \quad \tau = \frac{t}{L}\sqrt{\frac{E}{\rho_t}} \quad u_f = \sqrt{\frac{\rho_f}{E}}U_f \quad \psi = \bar{\psi}$$
$$f = \frac{EA_f}{EA} \quad \bar{\rho} = \frac{\rho_f}{\rho_t} \quad \bar{C} = \frac{Cl^2}{EA} \tag{12}$$
$$\bar{K}_w = \frac{K_w L^2}{EA} \quad \bar{G}_p = \frac{G_p}{EA} \quad \beta = \frac{K_s GA}{EA}$$

Substituting Eqs. (4), (7), (9) and (10) into Eq. (11), integrating by parts and setting the coefficients of $\delta U, \delta W$ and $\delta\psi$ to zero lead to the dimensionless equations of motion as

$$-\frac{1}{1-\upsilon^2}\frac{\partial^2 u_1}{\partial \xi^2} - \frac{1}{1-\upsilon^2}\frac{1}{\eta_1}\frac{\partial^2 w_1}{\partial \xi^2}\frac{\partial w_1}{\partial \xi} + (1+\bar{\rho}f_1)\frac{\partial^2 u_1}{\partial \tau^2} \tag{13}$$
$$-\sqrt{\bar{\rho}}f_1 u_f\frac{1}{\eta_1}\frac{\partial^2 w_1}{\partial \xi\partial \tau}\frac{\partial w_1}{\partial \xi} - f_1 u_f^2\frac{1}{\eta_1}\frac{\partial^2 w_1}{\partial \xi^2}\frac{\partial w_1}{\partial \xi}$$
$$-\bar{\mu}\sqrt{\bar{\rho}}f_1\frac{1}{\eta_1}\frac{\partial^3 u_1}{\partial \xi^2\partial \tau} + f_1\bar{\mu}u_f\left(\frac{1}{\eta_1}\right)^2\frac{\partial^3 w_1}{\partial \xi^3}\frac{\partial w_1}{\partial \xi} = 0$$

$$-\beta_1\frac{\partial^2 w}{\partial \zeta^2} - \eta\beta_1\frac{\partial \bar{\psi}}{\partial \zeta} - (1+\bar{\rho}f_1)\frac{1}{\eta_1}\frac{\partial^2 u}{\partial \tau^2}\frac{\partial w}{\partial \zeta} + \sqrt{\bar{\rho}}f_1 u_f\left(\frac{1}{\eta}\right)^2\frac{\partial^2 w}{\partial \zeta\partial \tau}\left(\frac{\partial w}{\partial \zeta}\right)^2$$
$$+ f_1 u_f^2\left(\frac{1}{\eta}\right)^2\frac{\partial^2 w}{\partial \zeta^2}\left(\frac{\partial w}{\partial \zeta}\right)^2 - \frac{1}{1-\upsilon^2}\frac{1}{\eta}\frac{\partial u}{\partial \zeta}\frac{\partial^2 w}{\partial \zeta^2} - \frac{1}{2}\frac{1}{1-\upsilon^2}\left(\frac{1}{\eta}\right)^2\left(\frac{\partial w}{\partial \zeta}\right)^2\frac{\partial^2 w}{\partial \zeta^2} \tag{14}$$

$$+(1+f_1\bar{\rho})\frac{\partial^2 w}{\partial\tau^2}+2\sqrt{\bar{\rho}}f_1u_f\frac{\partial^2 w}{\partial\zeta\partial\tau}-\sqrt{\bar{\rho}}f_1u_f\left(\frac{1}{\eta_1}\right)^2\frac{\partial^2 w}{\partial\zeta^2}\frac{\partial w}{\partial\zeta}\frac{\partial w}{\partial\tau}$$
$$-\sqrt{\bar{\rho}}f_1u_f\frac{1}{\eta}\frac{\partial^2 w}{\partial\zeta^2}\frac{\partial u}{\partial\tau}-\sqrt{\bar{\rho}}f_1u_f\frac{1}{\eta_1}\frac{\partial^2 u}{\partial\zeta\partial\tau}\frac{\partial w}{\partial\zeta}+f_1u_f^2\frac{\partial^2 w}{\partial\zeta^2}$$
$$-\sqrt{\bar{\rho}}f_1\bar{\mu}\frac{1}{\eta}\frac{\partial^3 w}{\partial\zeta^2\partial\tau}+\bar{\mu}\sqrt{\bar{\rho}}f_1\left(\frac{1}{\eta}\right)^2\frac{\partial^3 u}{\partial\zeta^2\partial\tau}\frac{\partial w}{\partial\zeta}^2-f_1u_f\bar{\mu}\left(\frac{1}{\eta}\right)^3\frac{\partial^3 w}{\partial\zeta^3}\left(\frac{\partial w}{\partial\zeta}\right)^2$$
$$-f_1u_f\bar{\mu}\left(\frac{1}{\eta}\right)^3\left(\frac{\partial^2 w}{\partial\zeta^2}\right)^2\frac{\partial w}{\partial\zeta}-f_1\bar{\mu}u_f\left(\frac{1}{\eta}\right)\frac{\partial^3 w}{\partial\zeta^3}+f_1\bar{\mu}u_f\left(\frac{1}{\eta}\right)^3\left(\frac{\partial^2 w}{\partial\zeta^2}\right)^2\frac{\partial w}{\partial\zeta}=0$$

$$\frac{E}{1-\upsilon^2}\bar{I}\left(\frac{1}{\eta}\right)^2\frac{\partial^2\bar{\psi}}{\partial\zeta^2}+\beta_1\frac{1}{\eta_1}\frac{\partial w}{\partial\zeta}$$
$$+\beta_1\bar{\psi}+(\bar{I}+\bar{\rho}_f\bar{I}_f)\left(\frac{1}{\eta}\right)^2\frac{\partial^2\bar{\psi}}{\partial\tau^2}=0 \qquad (15)$$

The associated boundary conditions can be expressed as

$$w=v=u=0 \quad @ \quad x=0,L$$
$$\frac{\partial w}{\partial x}=0 \quad @ \quad x=0,L \qquad (16)$$

3. Differential Quadrature Method

The differential quadrature (DQ) method is used to solve the nonlinear Eqs. (13)-(15) and the associated boundary conditions to determine the nonlinear free vibration frequencies of the pipes. The main idea of the differential quadrature (DQ) method is that the derivative of a function at a sample point can be approximated as a weighted linear summation of the function value at all of the sample points in the domain. The functions f = {u, w, ψ} and their k^{th} derivatives with respect to x can be approximated as [7]

$$\frac{d^n f(x_i)}{dx^n}=\sum_{j=1}^{N}C_{ij}^{(n)}f(x_j) \qquad n=1,\dots,N-1. \qquad (17)$$

where N is the total number of nodes distributed along the x-axis and C_{ij} is the weighting coefficients, the recursive formula for which can be found in [8]. The cosine pattern is used to generate the DQ point system

$$X_i=\frac{L}{2}\left[1-\cos\left(\frac{i-1}{N_x-1}\right)\pi\right] \qquad i=1,\dots,N \qquad (18)$$

Using DQM, Eqs. (13) to (15) can be expressed in matrix form as

$$\left(\left[\underbrace{K_L+K_{NL}}_{K}\right]+\Omega[C]+\Omega^2[M]\right)\begin{Bmatrix}\{d_b\}\\\{d_d\}\end{Bmatrix}=0, \qquad (19)$$

where M is the "mass" matrix, K_L is the linear "stiffness" matrix and K_{NL} is the nonlinear stiffness matrix.

4. Numerical Results

The final converged solutions using the numerical procedure outlined in section *B* above are illustrated as nolinear frequency and critical fluid velocity in Figs 2-6 below.

Fig. 2 and 3 illustrate the effects of aspect ratio (*L/R*) on the dimensionless frequency versus fluid velocity and nonlinear frequency ratio against maximum amplitude, respectively. It is evident that an increase in the aspect ratio increase dimensionless frequency and critical fluid velocity. Also, with increasing *L/R*, nonlinear frequency ratio increases. This is because increasing *L/R* leads to softer pipe.

Fig. 4 and 5 illustrate the influence of the normalized Pasternak shear modulus (*Kg*) on dimensionless frequency versus fluid velocity and nonlinear frequency ratio (i.e. the dimensionless nonlinear to linear frequency (Ω_{NL}/Ω_L) versus maximum amplitude (w_{max}), respectively. The result indicate that Ω_{NL}/Ω_L decreases substantially as harder elastic medium is employed. Hence, with increasing Pasternak shear modulus, Ω_{NL}/Ω_L decreases. Furthermore, as *Kg* increases, the critical fluid velocity and nonlinear frequency increase.

Fig. 6 illustrate the effect of fluid viscosity on the dimensionless frequency versus fluid velocity. The results indicating that viscous fluid increases natural frequency very little. However, during the flow of a fluid through a pipe as a Timoshenko beam, the effect of fluid viscosity on the vibration and instability of pipes may be ignored.

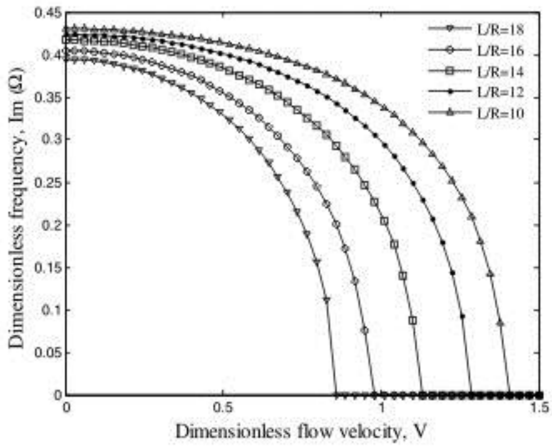

Fig. 2 The effect of geometrical parameter on nonlinear frequency versus Fluid velocity.

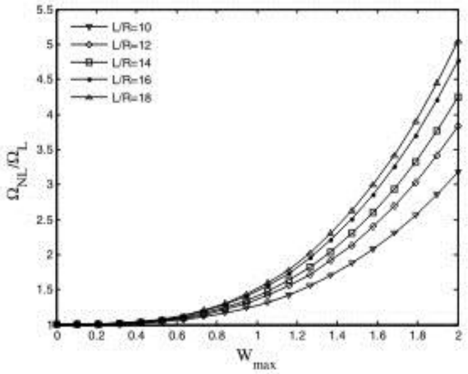

Fig. 3 The effect of geometrical parameter on nonlinear frequency ratio versus maximum amplitude.

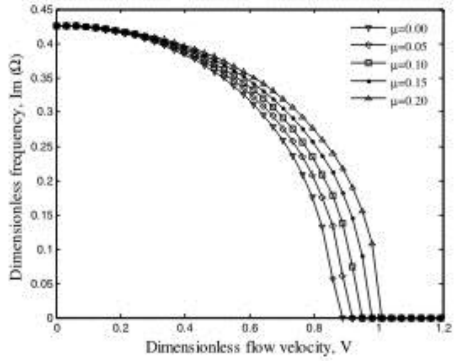

Fig. 6 The effect of fluid viscosity on nonlinear frequency versus fluid velocity.

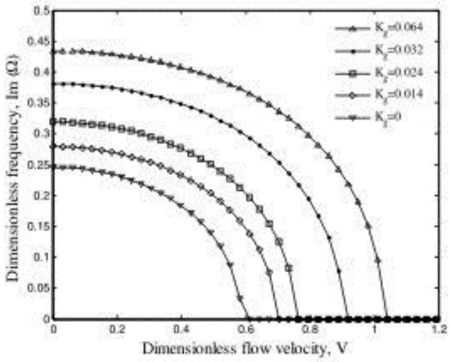

Fig. 4 The effect of Pasternak foundation on nonlinear frequency versus Fluid velocity.

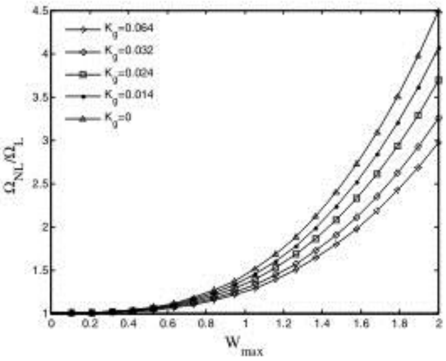

Fig. 5 The effect of Pasternak foundation on nonlinear frequency ratio versus maximum amplitude.

5. Conclusion

This paper investigates the nonlinear free vibration and instability of pipes based on von Karaman geometric nonlinearity and Timoshenko beam theory. The differential quadrature (DQ) method and a direct iterative approach are employed to obtain the nonlinear vibration frequencies and critical fluid velocity of pipe with clamped supported. Results indicate that the internal moving fluid plays an important role in the instability of the pipe. Furthermore, the nonlinear frequency and instability increases as the values of the elastic medium constants and viscosity of fluid increases.

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